Theoretical Investigation of Superradiant Lasing in 2- or 3- Level Atoms in an Optical Lattice

Gogyan A.[1,2], Kazakov G.[3], Bober M.[1], Zawada M.[1]

[1]Institute of Physics, Faculty of Physics, Astronomy and Informatics,
Nicolaus Copernicus University,
Toruń, Poland

[2]Institute for Physical Research, National Academy of Sciences of Armenia,
Ashtarak, Armenia

[3]Atominstitut, TU Wien, Vienna, Austria

***Abstract* — Active optical atomic clocks operating on narrow linewidth clock transitions based on a superradiant lasing are predicted to achieve precision orders of magnitude higher than any currently existing optical atomic clocks. We introduce a general theory of superradiant lasing and implement it for the examples of Ca and Sr atoms. Our simulations coincide well with the experimental results and explain some features of it. The presented model is valid for any two- and three-level system in an optical lattice.**

Keywords — active atomic clocks; bad cavity lasers;

I. Introduction

New emerging superradiant (SR) lasers on narrow "clock" transitions in alkaline-earth like atoms are under active consideration during the last years. They have been experimentally observed in several experiments recently, see e.g. [1,2,3], when the light is emitted collectively by several atoms at the same time. In contrast to the usual lasers the SR lasers operate in the so-called "bad-cavity" regime, when the cavity decay rate is much larger than the atom-cavity coupling coefficient and the laser operation does not rely on a large population of photons within the laser cavity to maintain the coherence. The coherence in such lasers is primarily stored in the gain medium, rather than the cavity field. Their gain is characterized by a much narrower linewidth than the optical cavity mode. In contrast to "conventional" good-cavity lasers, in bad-cavity lasers the fluctuations of cavity length --- the main limitation for short-term stability of conventional lasers [4] --- are highly suppressed.

Here we present the theory of superradiant pulse production in alkaline-earth like atoms in a cavity, a feasibility study has been performed [5] for an experimental implementation using the example of ^{40}Ca atoms, reported in [3]. Here we also present the model of superradiant pulse formation in ensembles of ^{87}Sr atoms, and analyse how decoherence and inhomogeneous broadening affect these pulses. Understanding of these effects is crucial for the development of continuously-operating superradiant lasers, particularly in the schemes proposed in [6]

II. Basic Equations

We consider simplified model of three-level atoms in a relatively low-finesse cavity of frequency ω_c and decay rate κ (Fig.1(a)) that is tuned in resonance with the lasing transition $2 \rightarrow 1$ (Fig. 1(b)). While interacting with the cavity mode the atoms coherently emit photons, giving rise to a SR pulse.

The SR pulse is described by the flux of the output field defined by

$$\frac{dn_{out}(t)}{dt} = \kappa\langle \hat{c}^{+}(t)\hat{c}(t)\rangle, \qquad (1)$$

where $n_{out}(t)$ is the mean photon number leaving the cavity with the rate κ through the mirror, $\hat{c}(t)$ is the annihilation operator of the cavity field. We neglect any collective effects of coupling of the atoms to bath modes. Also, we suppose that the atoms can be driven by an external laser field. Such a system is governed by the Born-Markov master equation.

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right] + \frac{\kappa}{2}\hat{\hat{L}}[\hat{c}]\hat{\rho} + \sum_{j=1}^{N}\sum_{i,k}\frac{R_{ik}}{2}\hat{\hat{L}}[\hat{\sigma}_{ik}^{j}]\hat{\rho}, \qquad (2)$$

where $\hat{\rho}$ is the density matrix for the whole system, $\hat{\hat{L}}[\hat{o}]\hat{\rho} = 2\hat{o}\hat{\rho}\hat{o}^{+} - \hat{o}^{+}\hat{o}\hat{\rho} - \hat{\rho}\hat{o}^{+}\hat{o}$ is the Lindbladian superoperator, operators $\hat{o}$ are called *jump operators*. The term $\frac{\kappa}{2}\hat{\hat{L}}[\hat{c}]\hat{\rho} = \frac{\kappa}{2}(2\hat{c}\hat{\rho}\hat{c}^{+} - \hat{c}^{+}\hat{c}\hat{\rho} - \hat{\rho}\hat{c}^{+}\hat{c})$ accounts for the loss of photons from the cavity with decay rate κ, and the last sum accounts for the processes with individual atoms,

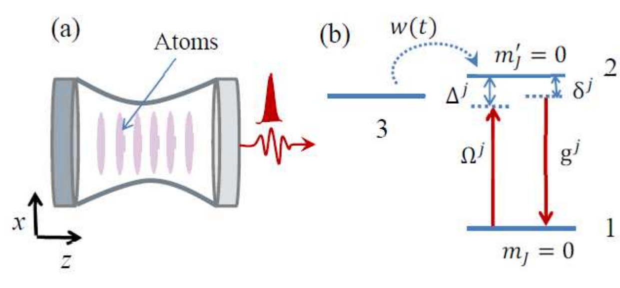

Fig. 1 (a) Magic wavelength lattice confines atoms in a cavity in lattice sites. Applied magnetic field is parallel to the x-axis, the SR pulse is generated along the cavity axis z. (b) Relevant level diagram of j-th atom, interacting with a clock transition laser field Ω^{j}, cavity mode g^{j}. Atoms are either initially prepared at the state 2 or are prepared at state 3 and incoherently pumped to state 2 by the rate $w(t)$.

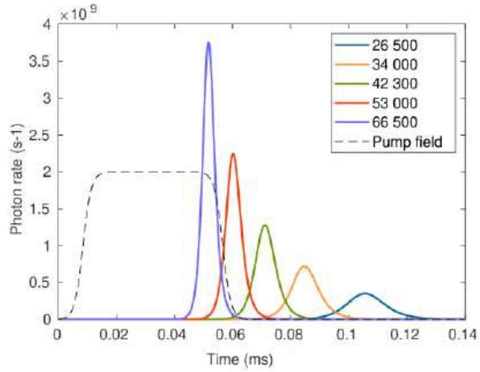

Fig. 2 Generated SR pulse flux for different values of atom number.

particularly, for the incoherent pumping from state 3 to state 2 (at $i = 3, k = 2$, $R_{32} = w$ is the repumping rate) and all the relaxations in the system, i.e. the spontaneous transitions between the atomic states ($i, k = 1,2,3$), and decoherence on the lasing transition with the rate γ_R, see [5] for more details. Summation is taken over all N atoms of the system.

The relevant expectation values of atomic and field operators follow the equation $\langle \dot{\hat{O}} \rangle = Tr[\dot{\rho}\, \hat{O}]$. In the mean-field approximation we factorize the atomic and field operators expectation values as $\langle \hat{\sigma}^j{}_{ik}\, \hat{c} \rangle = \langle \hat{\sigma}^j{}_{ik} \rangle \langle \hat{c} \rangle$, where $\hat{\sigma}^j{}_{ik}$ is the atomic projection operator $|i\rangle\langle k|$ for the j-th atom. This yields to the equation for the field operator:

$$\frac{d\langle \hat{c} \rangle}{dt} = -\left(\frac{\kappa}{2} + i\delta\right)\langle \hat{c} \rangle - i \sum_{j=1}^{N} N_j \langle \hat{\sigma}_{12}^{j} \rangle. \quad (3)$$

The Hamiltonian of the considered system of N atoms interacting with the cavity in the rotating wave approximation has the form

$$\hat{H} = \hbar \sum_{j=1}^{N} (\Delta_j\, \hat{\sigma}_{22}^{j} + g^j \hat{\sigma}_{21}^{j}\, \hat{c} + \Omega^j \hat{\sigma}_{21}^{j}) + \text{H.c.}, \quad (4)$$

where $\Delta_j = \omega_{21}^{j} - \omega_L$ is the detuning of frequency ω_L of the cavity mode from frequency ω_{21}^{j} of the lasing transition in j-th atom, Ω^j is the laser field Rabi frequency for the j-th atom. In general, g^j depends on the spatial coordinates of the j-th atom. Since the atoms move fast within the trap site, we average spatially the coupling coefficient

$$g^j = g(x, y, z) = g_0 \frac{w_c^2}{w_c^2 + \frac{w_r^2}{4}} \exp\left(-\frac{k^2 w_z^2}{8}\right) \cos k z_0, \quad (5)$$

where g_0 is the peak value of the coupling constant, z_0 is the axial position of the trap site, w_c is the cavity waist, w_r and w_z are the atomic cloud size at the trap site in the radial and axial directions, respectively, k is the wavenumber. The details of the calculations are presented in [5].

III. RESULTS

We numerically solve Eq. (2) along with the equations for the atomic projection operators and calculate the flux (1) using feasible experimental parameters. This allows us to find the temporal shape of the SR pulse, as well as to investigate and analyze its properties.

A. Formation of superradiant pulse: Calcium

For Ca atoms, we consider atoms initially prepared in the state 3, which, in general, can be any long-living upper-lying state, from which atoms can be pumped to the state ${}^3P_1, m_J' = 0$. The lasing transition $2 \to 1$ corresponds to the transition ${}^3P_1, m_J' = 0 \to {}^1S_0, m_J = 0$.

We take a cavity finesse of $F = 2200$ and a decay rate $\kappa = \pi \times 2260$ kHz. The cavity waist for the ${}^1S_0 \to {}^3P_1$ transition at 657 nm wavelength is $w_c = 190$ µm. We suppose the presence of an optical lattice with magic wavelength of 800.8 nm and with the depth of 20 MHz, which axial and radial frequencies are around 1.9 MHz and 1.8 kHz respectively.

The peak value of the pumping field Rabi frequency is $w_0 = 2\pi \times 25$ kHz and the duration is 50 µs. We assume that on the typical timescales of the SR pulse the collisional losses and dephasing do not have a considerable influence on the experiment.

Figure 2 shows the simulated SR pulse flux for atom numbers $N = 66500$, 53000, 42300, 34000, 26500. As seen from the figure, the pulse width decreases as the number of atoms is increased and at the same time the peak value of the SR increases. Parameters for the numerical calculations and the simulated results are the same as in [3], except the atomic numbers, which also were the fit parameters in [3].

The reason for this disagreement in the fitted number of atoms between our results and [4] is that in our model we take into account the mismatch between the nodes and antinodes of the magic wavelength lattice (800.8 nm) and the cavity mode at the lasing transition (657 nm), what leads to inhomogeneous atom-cavity coupling coefficient in different lattice sites.

B. Formation of single superradiant pulse: Strontium

Results of the simulations for ^{87}Sr atoms are presented in Figs. 3 and 4. In this case we considered atoms are initially at the state 2 and there is no pumping from the state 3 to 2. The states 1 and 2 correspond to the states ${}^1S_0, m = 9/2$ and ${}^3P_0, m = 9/2$, respectively. For our simulation we considered a system of $N = 10^5$ atoms interacting with the cavity field. The cavity has the mode waist $w_c = 100$ µm, length $L =$ 15cm, and the finesse of $F = 10^5$. It gives maximum coupling strength between the atoms and the cavity field $g_0 \simeq 19.2$ 1/s, and cavity decay rate $\kappa \simeq \pi \times 10$ kHz. Also, we introduce a parameter $\Gamma_c = \sum_{j=1}^{N} \frac{{g^j}^2}{\kappa} \simeq 2\pi \times 46.6$ Hz. Individual atomic detunings Δ^j are distributed uniformly between $-\Delta_0$ and $+\Delta_0$. Further we denote $\langle \hat{c} \rangle = E$ and participation M as a fraction of the atoms taking part in formation of the pulse.

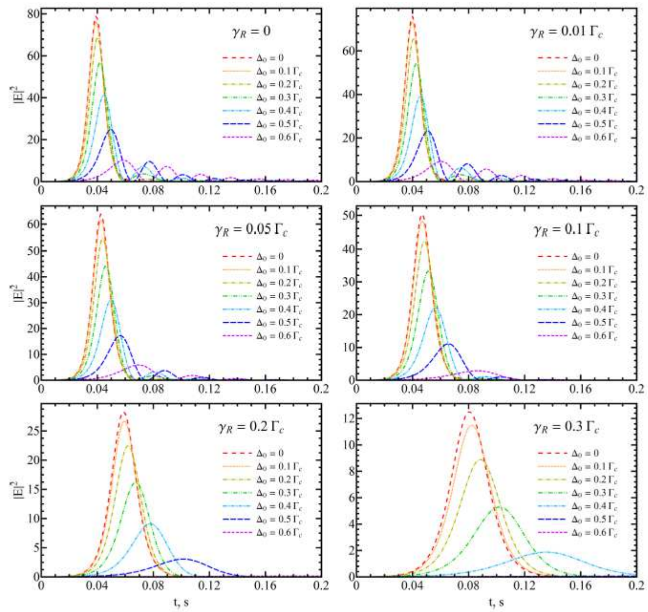

Fig. 3 Generated SR pulse flux for different decoherence rates and inhomogeneous widths.

Figure 3 represents simulated SR pulses in time domain for different decoherence rates γ_R and inhomogeneous widths Δ_0. One may note that, if $\Delta_0 \gg \gamma_R$, a chain of pulsations is generated instead of single SR pulse. Phases of the cavity field of these pulsations alternate. In the scheme with sequential delivery of the atomic ensembles into the cavity [6] such pulsations may violate the proper inheritance of the phase between atomic ensembles.

To compare the characteristics of the system with inhomogeneous broadening and inhomogeneous coupling with similar characteristics of the "homogeneous" system, we present the dependences of delay time t_d, duration Δt, peak intracavity photon number $|E|^2_{max}$ and participation M in units of the same quantities for homogenous broadening and coupling on Δ_0/Γ_c (Fig. 4) and $\Delta_0/\sqrt{\Gamma_c(\Gamma_c - 2\gamma_R)}$ (Fig. 5) for different γ_R. At Δ_0 (system without inhomogeneous broadening) delay time and duration of the superradiant pulse in the system with inhomogeneous coupling can be well described by the respective expressions for the system with homogeneous coupling, whereas the amplitude $|E|^2_{max}$ of the superradiant pulse and participation M for the system with inhomogeneous coupling are about 70% of the ones for the system with homogeneous coupling, see Fig. 4. These results agree with the ones presented in [7]. We see that the increase of inhomogeneous broadening Δ_0 leads to a decrease of amplitude of the superradiant pulse and of participation, and to an increase of duration and delay time. Moreover, the larger is γ_R, the faster are these tendencies. We observed, however, that normalized parameters behave approximately the same way as functions of $\Delta_0/\sqrt{\Gamma_c(\Gamma_c - 2\gamma_R)}$, see Fig. 5.

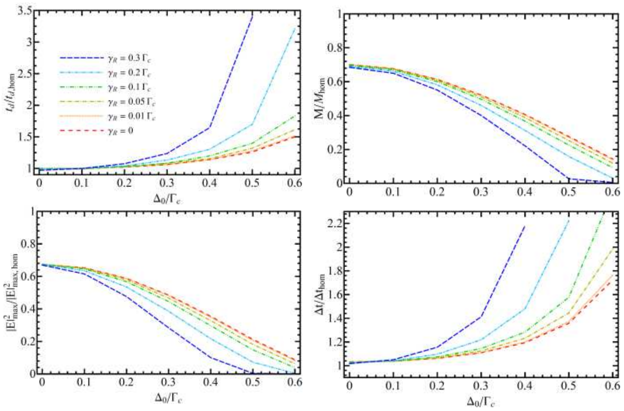

Fig. 4 Normalized delay time t_d, duration Δt, amplitude $|E|^2$ and participation M for simulated superradiant pulse in the system with inhomogeneous coupling and inhomogeneous broadening. For multiple pulsations, only the first pulsation is considered.

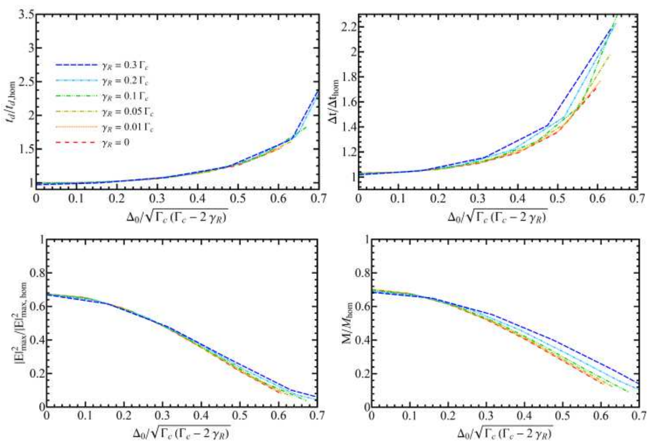

Fig. 5 Same as Fig. 4 as a function on $\Delta_0/\sqrt{\Gamma_c(\Gamma_c - 2\gamma_R)}$.

IV. Conclusions

Concluding, we have presented the general theory of superradiant lasing in 2- and 3-level atomic systems, taking into account incoherent pumping of atoms. We have implemented the model for the examples of Ca and Sr atoms. Simulations demonstrate good correspondence with experimental results and explain some observed features. Also, we studied the dependence of parameters of the SR pulse on the inhomogeneous broadening and decoherence rate in the presence of inhomogeneous coupling. Our models can be applied to any two- or three-level system in trapped alkaline-earth-like atoms in an optical lattice.

Acknowledgment

The "A next-generation worldwide quantum sensor network with optical atomic clocks" project is carried out within the TEAM IV Programme of the Foundation for Polish Science cofinanced by the European Union under the European Regional Development Fund.

The project EMPIR 15SIB03 OC18, 17FUN03 USOQS i 18SIB05 ROCIT, the EMPIR Programme cofinanced by the Participating States and from the European Union's Horizon 2020 Research and Innovation Programme. The European Union's Horizon 2020 Research and Innovation Programme under grant agreement No 820404 (iqClock project) and No 860579 (MoSaiQC). The National Science Centre, Poland, Project No 2017/25/B/ST2/00429.I The National Science Centre, Poland, under QuantEra programme No 2017/25/Z/ST2/03021.

REFERENCES

[1] M. A. Norcia, M. N. Winchester, J. R. K. Cline, and J. K. Thompson, “Superradiance on the millihertz linewidth strontium clock transition,” Sci. Adv. 2(10), 2016, p. e1601231.

[2] S. A. Schäffer, M. Tang, M. R. Henriksen, A. A. Jørgensen, B. T. R. Christensen, and J. W. Thomsen, “Lasing on a narrow transition in a cold thermal strontium ensemble,” Phys. Rev. A 101(1), 2020, p. 013819.

[3] T. Laske, H. Winter, and A. Hemmerich, “Pulse delay time statistics in a superradiant laser with calcium atoms,” Phys. Rev. Lett. 123(10), 2019, p. 103601.

[4] K. Numata, A. Kemery, and J. Camp, “Thermal-noise limit in the frequency stabilization of lasers with rigid cavities,” Phys. Rev. Lett. 93(25), 2004, p. 250602.

[5] A. Gogyan, G. Kazakov, M. Bober, and M. Zawada, “Characterisation and feasibility study for superradiant lasing in ^{40}Ca atoms”, Opt. Express 28, 2020, pp. 6881-6892.

[6] G. A. Kazakov and T. Schumm, “Active optical frequency standard using sequential coupling of atomic ensembles,” Phys. Rev. A 87(1), 2013, p. 013821.

[7] M. A. Norcia, “New tools for precision measurement and quantum science with narrow linewidth optical transitions”, PhD thesis, Department of Physics, University of Colorado, 2018.

.